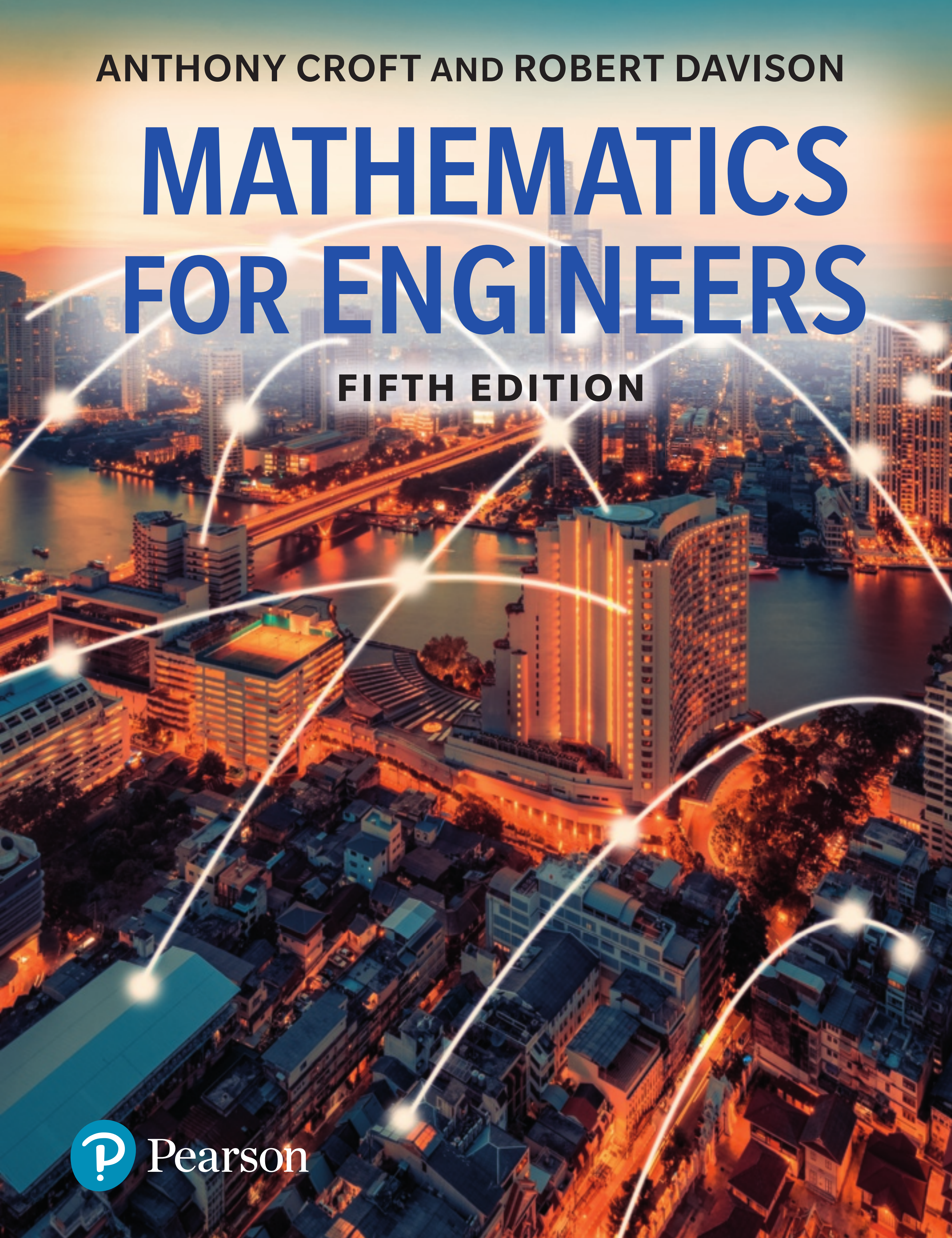
ANTHONY CROFT AND ROBERT DAVISON

MATHEMATICS FOR ENGINEERS

FIFTH EDITION



Pearson

Mathematics for Engineers

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Fifth Edition

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To Kate and Harvey (AC)
To Kathy (RD)

Brief contents

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Companion Website

ON THE WEBSITE

For open-access student resources specifically written to complement this textbook and support your learning, please visit **www.pearsoned.co.uk/croft**

Lecturer Resources

For password-protected online resources tailored to support the use of this textbook in teaching, please visit **www.pearsoned.co.uk/croft**

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Preface

Audience

This book has been written to serve the mathematical needs of students engaged in a first course in engineering or technology at degree level. Students of a very wide range of these programmes will find that the book contains the mathematical methods they will meet in a first-year course in most UK universities. So the book will satisfy the needs of students of aeronautical, automotive, chemical, civil, electronic and electrical, systems, mechanical, manufacturing, and production engineering, and other technological fields. Care has been taken to include illustrative examples from these disciplines where appropriate.

Aims

There are two main aims of this book.

Firstly, we wish to provide a readable, accessible and student-friendly introduction to mathematics for engineers and technologists at degree level. Great care has been taken with explanations of difficult concepts, and wherever possible statements are made in everyday language, as well as symbolically. It is the use of symbolic notation that seems to cause many students problems, and we hope that we have gone a long way to alleviate such problems.

Secondly, we wish to develop in the reader the confidence and competence to handle mathematical methods relevant to engineering and technology through an interactive approach to learning. You will find that the book encourages you to take an active part in the learning process – this is an essential ingredient in the learning of mathematics.

The structure of this book

The book has been divided into 24 chapters. Each chapter is subdivided into a unit called a **block**. A block is intended to be a self-contained unit of study. Each block has a brief introduction to the material in it, followed by explanations, examples and applications. Important results and key points are highlighted. Many of the examples require you to participate in the problem-solving process, so you will need to have pens or pencils, scrap paper and a scientific calculator to hand. We say more about this aspect below. Solutions to these examples are all given alongside.

Each block also contains a number of practice exercises, and the solutions to these are placed immediately afterwards. This avoids the need for searching at the back of the book for solutions. A further set of exercises appears at the end of each block.

At the end of each chapter you will find end of chapter exercises, which are designed to consolidate and draw together techniques from all the blocks within the chapter.

Some sections contain computer or calculator exercises. These are denoted by the computer icon. It is not essential that these are attempted, but those of you with access to graphical calculators or computer software can see how these modern technologies can be used to speed up long and complicated calculations.

Learning mathematics

In mathematics almost all early building blocks are required in advanced work. New ideas are usually built upon existing ones. This means that, if some early topics are not adequately mastered, difficulties are almost certain to arise later on. For example, if you have not mastered the arithmetic of fractions, then you will find some aspects of algebra confusing. Without a firm grasp of algebra you will not be able to perform the techniques of calculus, and so on. It is therefore essential to try to master the full range of topics in your mathematics course and to remedy deficiencies in your prior knowledge.

Learning mathematics requires you to participate actively in the learning process. This means that in order to get a sound understanding of any mathematical topic it is essential that you actually perform the calculations yourself. You cannot learn mathematics by being a spectator. *You* must use *your* brain to solve the problem, and *you* must write out the solution. These are essential parts of the learning process. It is not sufficient to watch someone else solve a similar problem, or to read a solution in a book, although these things of course can help. The test of real understanding and skill is whether or not you can do the necessary work on your own.

How to use this book

This book contains hundreds of fully worked examples. When studying such an example, read it through carefully and ensure you understand each stage of the calculation.

A central feature of the book is the use of interactive examples that require the reader to participate actively in the learning process. These examples are indicated by the pencil icon. Make sure you have to hand scrap paper, pens or pencils and a calculator. Interactive examples contain 'empty boxes' and 'completed boxes'. An empty box indicates that a calculation needs to be performed by you. The corresponding completed box on the right of the page contains the calculation you should have performed. When working through an interactive example, cover up the completed boxes, perform a calculation when prompted by an empty box, and then compare your work with that contained in the completed box. Continue in this way through the entire example. Interactive examples provide some help and structure while also allowing you to test your understanding.

Sets of exercises are provided regularly throughout most blocks. Try these exercises, always remembering to check your answers with those provided. Practice enhances understanding, reinforces the techniques, and aids memory. Carrying out a large number of exercises allows you to experience a greater variety of problems, thus building your expertise and developing confidence.

Content

The content of the book reflects that taught to first-year engineering and technology students in the majority of UK universities. However, particular care has been taken to develop algebraic skills from first principles and to give students plenty of opportunity to practise using these. It is our firm belief, based on recent experience of teaching engineering undergraduates, that many will benefit from this material because they have had insufficient opportunity in their previous mathematical education to develop such skills fully. Inevitably the choice of contents is a compromise, but the topics covered were chosen after wide consultation coupled with many years of teaching experience. Given the constraint of space we believe our choice is optimal.

Use of modern IT aids

One of the main developments in the teaching of engineering mathematics in recent years has been the widespread availability of sophisticated computer software and its adoption by many educational institutions. Once a firm foundation of techniques has been built, we would encourage its use, and so we have made general references at several points in the text. In addition, in some blocks we focus specifically on two common packages (Matlab and Maple), and these are introduced in the 'Using mathematical software packages' section on page xx. Many features available in software packages can also be found in graphical calculators.

On pages xxiii–xxiv we provide a reference table of Maple and Matlab commands that are particularly useful for exploring and developing further the topics in this book.

Additions for the fifth edition

We have been delighted with the positive response to *Mathematics for Engineers* since it was first published in 1998. In writing this fifth edition we have been guided and helped by the numerous comments from both staff and students. For these comments we express our thanks.

This fifth edition has been enhanced by the addition of numerous examples from even wider fields of engineering. Applicability lies at the heart of engineering mathematics. We believe these additional examples serve to reinforce the crucial role that mathematics plays in engineering. We hope that you agree.

Following useful suggestions from reviewers we have added new sections to cover the equation of a circle, locus of a point in the complex plane and solution of partial differential equations. We have enhanced and integrated the use of software in the solution of engineering problems.

We hope the book supports you in your learning and wish you every success.

Anthony Croft and Robert Davison
May 2018

Using mathematical software packages

One of the main developments influencing the learning and teaching of engineering mathematics in recent years has been the widespread availability of sophisticated computer software and its adoption by many educational institutions.

As engineering students, you will meet a range of software in your studies. It is also highly likely that you will have access to specialist mathematical software. Two software packages that are particularly useful for engineering mathematics, and which are referred to on occasions throughout this book, are Matlab and Maple. There are others, and you should enquire about the packages that have been made available for your use. A number of these packages come with specialist tools for subjects such as control theory and signal processing, so you will find them useful in other subjects that you study.

Common features of all these packages include:

- the facility to plot two- and three-dimensional graphs;
- the facility to perform calculations with symbols (e.g. a^2, $x + y$, as opposed to just numbers) including the solution of equations.

In addition, some packages allow you to write computer programs of your own that build upon existing functionality, and enable the experienced user to create powerful tools for the solution of engineering problems.

The facility to work with symbols, as opposed to just numbers, means that these packages are often referred to as **computer algebra systems** or **symbolic processors**. You will be able to enter mathematical expressions, such as $(x + 2)(x - 3)$ or $\dfrac{t - 6}{t^2 + 2t + 1}$, and subject them to all of the common mathematical operations: simplification, factorisation, differentiation, integration, and much more. You will be able to perform calculations with vectors and matrices. With experience you will find that lengthy, laborious work can be performed at the click of a button.

The particular form in which a mathematical problem is entered – that is, the **syntax** – varies from package to package. Raising to a power is usually performed using the symbol ^. Some packages are menu driven, meaning that you can often select symbols from a menu or toolbar. At various places in the text we have provided examples of this for illustrative purposes. This textbook is not intended to be a manual for any of the packages described. For thorough details you will need to refer to the manual provided with your software or its on-line help.

At first sight you might be tempted to think that the availability of such a package removes the need for you to become fluent in algebraic manipulation and other mathematical techniques. We believe that the converse of this is true. These packages are sophisticated, professional tools and as such require the user to have a good understanding of the functions they perform, and particularly their limitations. Furthermore, the results provided by the packages can be presented in a variety of forms (as you will see later in the book), and only with a thorough understanding of the mathematics will you be able to appreciate different, yet correct, equivalent forms, and distinguish these from incorrect output.

Figure 1 shows a screenshot from Maple in which we have defined the function $f(x) = x^2 + 3x - 2$ and plotted part of its graph. Note that Maple requires the following particular syntax to define the function: $f: = x \rightarrow x^2 + 3x - 2$. The quantity x^2 is input as x^2.

Finally, Figure 2 shows a screenshot from the package Matlab. Here the package is being used to obtain a three-dimensional plot of the surface $z = \sin(x^2 + y^2)$ as described in Chapter 21. Observe the requirement of Matlab to input x^2 as $x \cdot$^2.

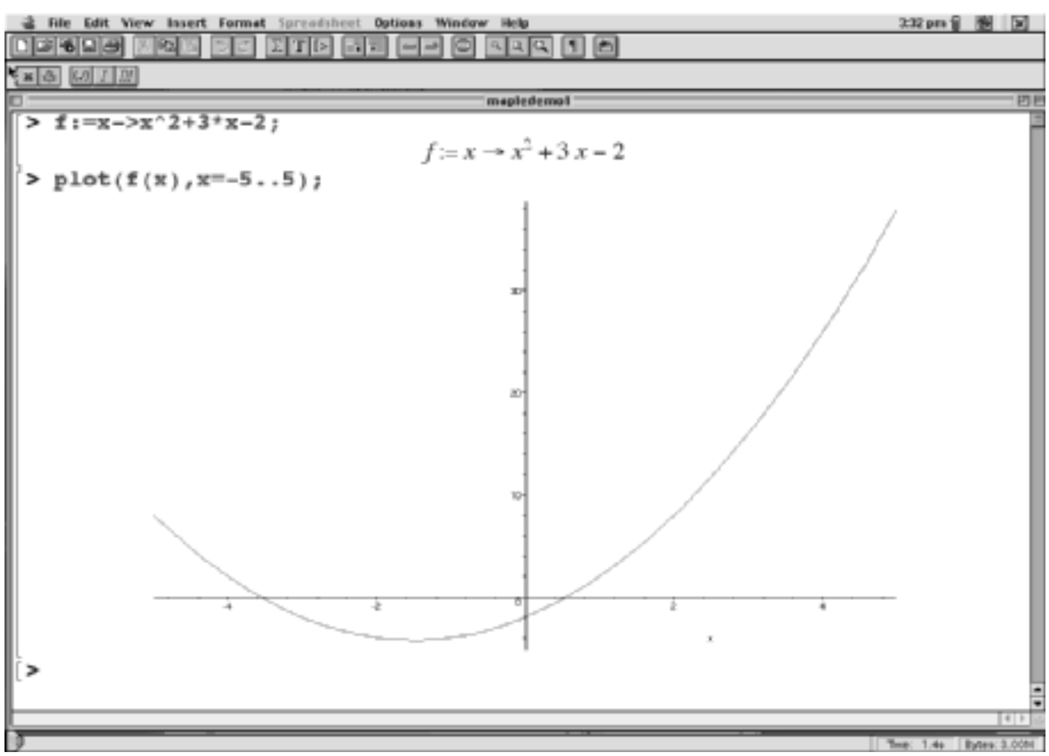

Figure 1 A screenshot from Maple showing the package being used to define the function $f(x) = x^2 + 3x - 2$ and plot its graph.

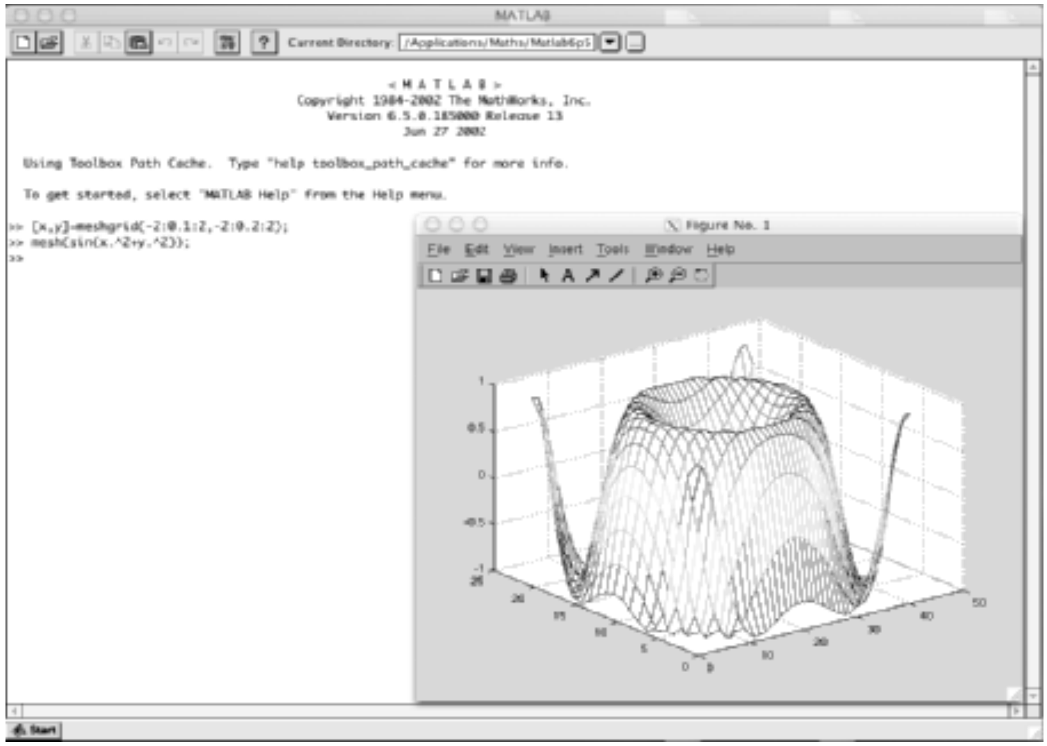

Figure 2
A screenshot from Matlab showing the package being used to plot a three-dimensional graph.

Where appropriate we would encourage you to explore the use of packages such as these. Through them you will find that whole new areas of engineering mathematics become accessible to you, and you will develop skills that will help you to solve engineering problems that you meet in other areas of study and in the workplace.

Useful mathematical software commands used throughout the book

The following commands are indicative only and should be read in conjuction with the software's on-line help and the examples found later in the book.

Purpose	Maple example	Matlab example	Page
Test whether an integer, n, is prime	`isprime(n)`	`isprime(n)`	11
Produce a prime factorisation of an integer, n	`ifactor(n)`	`factor(n)`	12
Plot graph of $y = f(x)$	`plot(x^3,x=-3..3,y=-20..20);`	`x=-3:0.1:3; y=x.^3;` `plot(x,y);`	152

Finding partial fractions expansion	`convert(x/(x^2+3*x+2),`
	`parfrac);`
		`n = [1];`
		`d =[1 3 2];`
		`[r,p,k] = residue(n,d)`
			279

Complex numbers	`use I` `(1+3*I)/(2-I)`	`use i or j` `(1+3*j)/(2-j)`	461
Find roots of a polynomial	`solve(s^3+s^2+s+1=0)`	`roots([1 1 1 1])`	462
Defining matrices	`A:= Matrix([[1,2,3],` `[4,5,6],[7,8,9]])`	`A = [1 2 3; 4 5 6; 7 8 9]`	530
Eigenvalues and eigenvectors	`Eigenvalues(A)` `Eigenvectors(A)`	`[V, D] = eig(A)`	641

Vectors: scalar and vector products	`with(LinearAlgebra);`
	`a:= Vector[row]([1,-2,3]);`
	`b:= Vector[row]([2,-1,1]);`
	`DotProduct(a,b);`
	`CrossProduct(a,b);`
		`a=[1 -2 3]`
		`b=[2 -1 1]`
		`dot(a,b)`
		`cross(a,b)`
			724

(*Continued*)

Purpose	Maple example	Matlab example	Page

First and higher derivatives	`f:=t-> t^2*sin(3*t);`
	`D(f)(t);      or diff(f(t),t);`
	`D(D(f))(t);   or diff(f(t),t,t);`
		`syms f(t)`
		`f(t) = t^2*sin(3*t)`
		`y = diff(f(t))`
		`z = diff(f(t),2)`
			766

Indefinite and definite integration	`int(x*cos(x)^2,x)`
	`int(1/t,t=1..2)`
		`syms x t`
		`int(x*cos(x)^2,x)`
		`int(1/t,1,2)`
			857

Differential equations with or without conditions	`dsolve(diff(y(x),x) - x*y(x)=0);`
	`dsolve({diff(y(x),x) - x*y(x)=0,y(0)=3})`
		`dsolve('Dy-x*y=0','x')`
		`dsolve('Dy-x*y=0',`
		`'y(0)=3','x')`
			1011

Sums of series	`sum(1/k,k=1..10);`	`sym k` `symsum(1/k,k,1,10)`	950

Taylor series	`taylor(sqrt(x),x=4,4);`	`taylor(sqrt(x),`
		`'ExpansionPoint',4,`
		`'Order',4)`
			972

3d plots	`plot3d(x^2+y^2,x=-2..2,y=-2..2);`	`[x,y]=meshgrid(-2:0.1:2,`
		`-2:0.1:2); z = x.^2+y.^2;`
		`mesh(z);`
			1059

Laplace transform	`with(inttrans):`
	`f:=t->t^2;`
	`laplace(f(t),t,s);`
		`syms t s`
		`f=t^2`
		`laplace(f,t,s)`
			1100

Fourier transform	`with(inttrans);`
	`f:=t->Heaviside(t)*exp(-t);`
	`fourier(f(t),t,w);`
		`syms t w`
		`f = heaviside(t)*exp(t)`
		`fourier(f,t,w)`
			1235

Chapter 1

Arithmetic

This chapter reminds the reader of the arithmetic of whole numbers. Arithmetic is the study of numbers. A mastery of numbers and the ways in which we manipulate and operate on them is essential. This mastery forms the bedrock for further study in the field of algebra.

Block 1 introduces some essential terminology and explains rules that determine the order in which operations must be performed. Block 2 focuses on prime numbers. These are numbers that cannot be expressed as the product of two smaller numbers.

Computers are used extensively in all engineering disciplines to perform calculations. Some of the examples provided in this book make use of the software packages Maple and Matlab which are commonly available for use in academic and industrial settings.

Because Maple and Matlab, in common with many similar packages, are designed to compute not just with single numbers but with entire sequences of numbers at the same time, data are sometimes entered in the form of arrays, as we will demonstrate. Arrays are multi-dimensional objects. Two particular types of array are **vectors** and **matrices,** which are studied in detail in Chapters 12–14.

Chapter 1 contents

BLOCK 1

Operations on numbers

1.1 Introduction

Whole numbers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$. Whole numbers are also referred to as **integers**. The **positive integers** are $1, 2, 3, 4, \ldots$. The **negative integers** are $\ldots, -4, -3, -2, -1$. The ... indicates that the sequence of numbers continues indefinitely. The number 0 is an integer but it is neither positive nor negative.

Given two or more whole numbers it is possible to perform an **operation** on them. The four arithmetic operations are addition (+), subtraction (−), multiplication (×) and division (÷).

Addition (+)

We say that $4 + 5$ is the **sum** of 4 and 5. Note that $4 + 5$ is equal to $5 + 4$ so that the order in which we write down the numbers does not matter when we are adding them. Because the order does not matter, addition is said to be **commutative**. When more than two numbers are added, as in $4 + 8 + 9$, it makes no difference whether we add the 4 and 8 first to get $12 + 9$, or whether we add the 8 and 9 first to get $4 + 17$. Whichever way we work we shall obtain the same result, 21. This property of addition is called **associativity**.

Subtraction (−)

We say that $8 - 3$ is the **difference** of 8 and 3. Note that $8 - 3$ is not the same as $3 - 8$ and so the order in which we write down the numbers is important when we are subtracting them. Subtraction is not commutative. Adding a negative number is equivalent to subtracting a positive number; thus $5 + (-2) = 5 - 2 = 3$. Subtracting a negative number is equivalent to adding a positive number: thus $7 - (-3) = 7 + 3 = 10$.

Key point

Adding a negative number is equivalent to subtracting a positive number.
Subtracting a negative number is equivalent to adding a positive number.

Multiplication (×)

The instruction to multiply the numbers 6 and 7 is written 6×7. This is known as the **product** of 6 and 7. Sometimes the multiplication sign is missed out altogether and we write (6)(7). An alternative and acceptable notation is to use a dot to represent multiplication and so we could write $6 \cdot 7$, although if we do this care must be taken not to confuse this multiplication dot with a decimal point.

Note that (6)(7) is the same as (7)(6) so multiplication of numbers is **commutative**. If we are multiplying three numbers, as in $2 \times 3 \times 4$, we obtain the same result if we multiply the 2 and the 3 first to get 6×4, as if we multiply the 3 and the 4 first to get 2×12. Either way the result is 24. This property of multiplication is known as **associativity**.

Recall that when multiplying positive and negative numbers the sign of the result is given by the following rules:

Key point

(positive) $\times$ (positive) = positive
(positive) $\times$ (negative) = negative
(negative) $\times$ (positive) = negative
(negative) $\times$ (negative) = positive

For example, $(-4) \times 5 = -20$ and $(-3) \times (-6) = 18$.

Division (÷)

The quantity $8 \div 4$ means 8 divided by 4. This is also written as $8/4$ or $\frac{8}{4}$ and is known as the **quotient** of 8 and 4. We refer to a number of the form p/q when p and q are whole numbers as a **fraction**. In the fraction $\frac{8}{4}$ the top line is called the **numerator** and the bottom line is called the **denominator**. Note that $8/4$ is not the same as $4/8$ and so the order in which we write down the numbers is important. Division is not commutative. Division by 0 is never allowed: that is, the denominator of a fraction must never be 0. When dividing positive and negative numbers recall the following rules for determining the sign of the result:

Key point

$$\frac{\text{positive}}{\text{positive}} = \text{positive}$$
$$\frac{\text{positive}}{\text{negative}} = \text{negative}$$
$$\frac{\text{negative}}{\text{positive}} = \text{negative}$$
$$\frac{\text{negative}}{\text{negative}} = \text{positive}$$

Example 1.1

Evaluate

(a) the sum of 9 and 4
(b) the sum of 9 and -4
(c) the difference of 6 and 3

(d) the difference of 6 and -3
(e) the product of 9 and 3
(f) the product of -9 and 3
(g) the product of -9 and -3
(h) the quotient of 10 and 2
(i) the quotient of 10 and -2
(j) the quotient of -10 and -2

Solution

(a) $9 + 4 = 13$
(b) $9 + (-4) = 9 - 4 = 5$
(c) $6 - 3 = 3$
(d) $6 - (-3) = 6 + 3 = 9$
(e) $9 \times 3 = 27$
(f) $(-9) \times 3 = -27$
(g) $(-9) \times (-3) = 27$
(h) $\frac{10}{2} = 5$
(i) $\frac{10}{-2} = -5$
(j) $\frac{-10}{-2} = 5$

Example 1.2 Reliability Engineering – Time between breakdowns

Reliability engineering is concerned with managing the risks associated with breakdown of equipment and machinery, particularly when such a breakdown is life-critical or when it can have an adverse effect on business. In Chapter 23 we will discuss the Poisson probability distribution which is used to model the number of breakdowns occurring in a specific time interval. Of interest to the reliability engineer is both the average number of breakdowns in a particular time period and the average time between breakdowns. The **breakdown rate** is the number of breakdowns per unit time.

Suppose a reliability engineer monitors a piece of equipment for a 48-hour period and records the number of times that a safety switch trips. Suppose the engineer found that there were three trips in the 48-hour period.

(a) Assuming that the machine can be restarted instantly, calculate the average time between trips. This is often referred to as the **inter-breakdown** or **inter-arrival time**.
(b) Calculate the breakdown rate per hour.

Solution

(a) With three trips in 48 hours, on average, there will be one trip every 16 hours. Assuming that the machine can be restarted instantly, the average time between trips is 16 hours. This is the inter-breakdown time.
(b) In 16 hours there is one trip. This is equivalent to saying that the breakdown rate is $\frac{1}{16}$ of a trip per hour.

More generally,

$$\text{the breakdown rate} = \frac{1}{\text{the inter-breakdown time}}$$